Government of Western Australia
Curriculum Council

WACE
Western Australian Certificate of Education

MATHEMATICS

2C/2D

Calculator-assumed

WACE Examination 2010

Final Marking Key

This 'stand alone' version of the WACE Examination 2010 Final Marking Key is provided on an interim basis.

The Standards Guide for this examination will include the examination questions, marking key, question statistics and annotated candidate responses. When the Standards Guide is published, this document will be removed from the website.

Question 8 **(3 marks)**

Using your calculator, determine the value of $a^2\sqrt{b}$, where $a = 1387 \times 10^4$ and $b = 9.203 \times 10^{-2}$.

(a) Write down the answer given by your calculator. (1 mark)

Solution
5.836031578 E+13 (Casio); 5.83603157829 E+13(TI); 5.8360315783 E+13 (HP)
Specific Behaviours
✓ states correct answer from calculator

(b) Write the answer in scientific notation, correct to three significant figures. (2 marks)

Solution
5.84×10^{13}
Specific Behaviours
✓ expresses answer in scientific notation ✓ expresses answer correctly to three significant figures

Question 9 **(5 marks)**

Tax rates 2009–10

Taxable income	Tax on this income
$0 – $6000	Nil
$6001 – $35 000	15c for each $1 over $6 000
$35 001 – $80 000	$4350 plus 30c for each $1 over $35 000
$80 001 – $180 000	$17 850 plus 38c for each $1 over $80 000
$180 001 and over	$55 850 plus 45c for each $1 over $180 000

The table above, from the Australian Taxation Office website, shows the tax rates for the 2009–10 financial year.

Luke has a taxable income of $93 874.

(a) Calculate the amount of tax Luke will be required to pay. (3 marks)

Solution
17 850 + .38 (13 874) = 17 850 + 5 272.12 = $23 122.12
Specific Behaviours
✓ identifies correct tax bracket (38 cents) ✓ determines variable tax amount (13 874) ✓ calculates correct total amount (includes fixed amount) Ans only 2 marks

(b) Louise, Luke's partner, has stayed at home throughout the 2009–10 financial year to take care of their young child. For the 2010–11 financial year, they decide to job share, with each earning half of Luke's previous full-time taxable income.

Assuming tax rates stay the same, what effect would this 2010–11 arrangement have on the total amount of tax paid by both Luke and Louise compared with that paid by Luke for the 2009–10 financial year? (2 marks)

Solution
Tax on $46 937.00 = 4 350 + 0.30 (11 937) = $7 931.10 Total tax paid by couple = 2 x 7 931.00 = $15 862.20 Which is $7 259.92 less than then amount of tax paid if only Luke worked full time.
Specific Behaviours
✓ calculates total tax paid by couple ✓ calculates the difference in tax amount or states that tax decreases

Question 10 **(6 marks)**

(a) Find the next two terms for each of the sequences defined below.

(i) $T_{n+1} = 0.5T_n$ $T_1 = 40$ (2 marks)

Solution
$T_2 = 0.5T_1 = 20$ $T_3 = 0.5T_2 = 10$
Specific Behaviours
✓ calculation of T_2 ✓ calculation of T_3

(ii) $T_{n+1} = T_{n+2} - T_n$ $T_1 = 1,\ T_2 = 1$ (2 marks)

Solution
$T_3 = T_2 + T_1 = 2$ $T_4 = T_3 + T_2 = 3$
Specific Behaviours
✓ calculation of T_3 ✓ calculation of T_4

(b) Write a recursive rule for the sequence of numbers 8, 12, 18, 27, ... (2 marks)

Solution
$T_{n+1} = \frac{3}{2} T_n$, $T_1 = 8$
Specific Behaviours
✓ recognising the rule ✓ states value of first term.

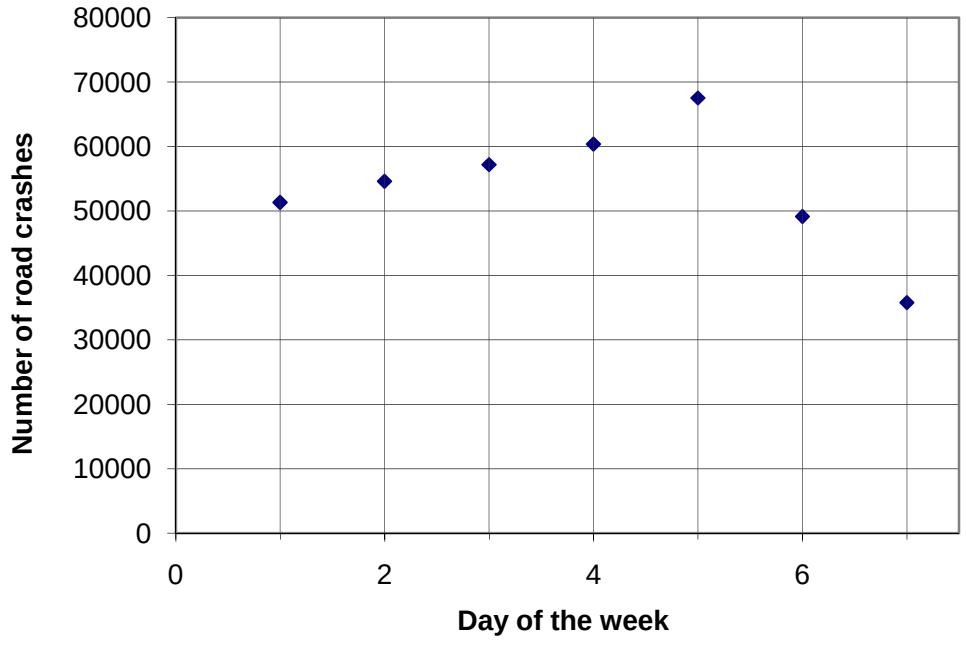
Question 11 **(10 marks)**

The table and graph show data on the number of road crashes in Western Australia in the 10-year period 1995–2004, by days of the week.

Number of road crashes in Western Australia 1995–2004, by days of the week

	Day	Number of crashes (nearest thousand)
Monday	1	51 000
Tuesday	2	55 000
Wednesday	3	57 000
Thursday	4	60 000
Friday	5	68 000
Saturday	6	49 000
Sunday	7	36 000

Number of road crashes in Western Australia 1995–2004, by days of the week



(a) Based on the data, which is the safest day of the week on the roads in Western Australia? (1 mark)

Solution
Sunday
Specific Behaviours
✓ identifies correct day

(b) Describe any trends in terms of the variables. (3 marks)

Solution
The number of road crashes by days of the week in Western Australia for the period 1995–2004 shows an increasing trend from Monday through to Friday and a decreasing trend from Friday through to Sunday.
Specific Behaviours
✓ identifies increasing trend (Monday to Friday) ✓ identifies decreasing trend (Friday to Sunday) ✓ relates variables to trends

(c) If a road crash is selected at random from the data for investigation, what is the probability that it occurred on a Saturday or Sunday? (2 marks)

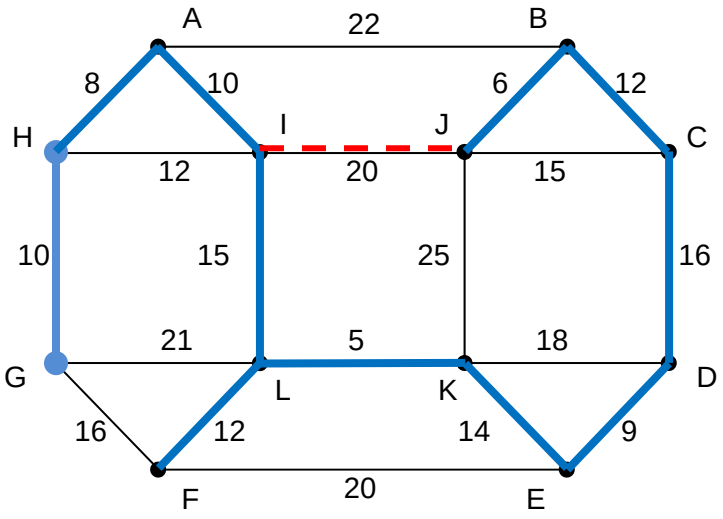
Solution
$\frac{49000+36000}{376000}=\frac{85}{376}$ Or answer approximately 0.2261 (accept: , 0.22 or 0.226)
Specific Behaviours
✓ uses correct values from table ✓ calculation of probability

(d) Suppose 1000 of the road crashes are selected at random from the data, and it is found that 223 of them occurred on either a Saturday or a Sunday. Is the result of 223 within the range of values that you would expect? Justify your answer. (4 marks)

Solution
Expected number of crashes (based on the probability $\frac{85}{376}$) $=\frac{1000\times 85}{376}$ = 226 As 223 is close to 226, the result is likey.
Specific Behaviours
✓ applies correct probability value to estimate number of crashes ✓ calculates expected number of crashes ✓ recognises ‘close to 226’ ✓ concludes that the result is likely

Question 12 **(6 marks)**

A communication network linking 12 centres A, B, C, …, K, L is to be constructed. The possible connections are given in the following network. The number on each arc represents the cost, in thousands of dollars, of establishing the connection.



To minimise the cost, the network management decides to use a minimal spanning tree solution.

(a) Indicate clearly the minimal spanning tree solution on the network above. (3 marks)

Solution
Shown Above
Specific Behaviours
Carries through to a degree of accuracy ✓ shows at least 7 correct connections, with no cycle evident ✓ shows at least 9 correct connections, with no cycle evident ✓ shows all connections correctly.

(b) Determine the minimum cost of constructing the communication network. (1 mark)

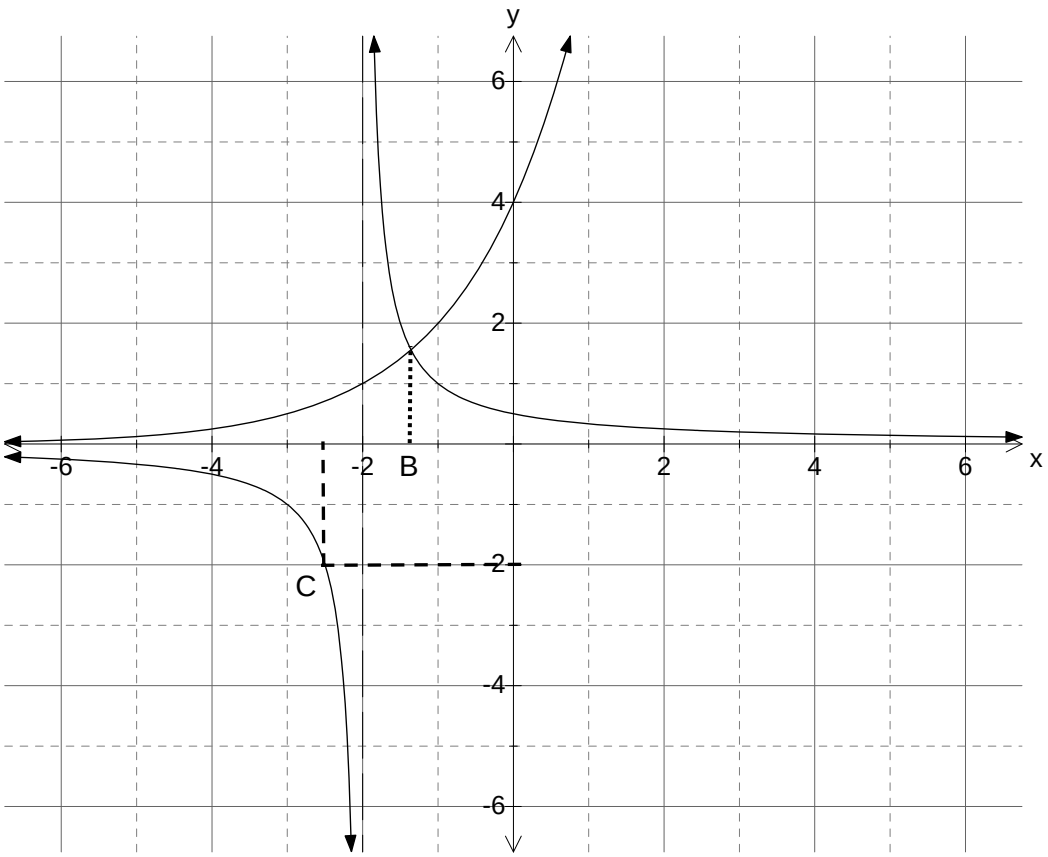
Solution
$117 000
Specific Behaviours
✓ expresses total cost in thousands of dollars (based on result in (a))

(c) The cost of connecting I and J has been overestimated by $6 000. How does this information change the minimum cost of constructing the network? (2 marks)

Solution
Adding IJ (at a cost of 14 000) and deleting CD (at a saving of 16 000) results in a minimum spanning tree costing $115 000. Saves $2 000.
Specific Behaviours
✓ identifies connections affected by changed conditions.
✓ describes effect of changed conditions. Decrease only 0

Question 13 **(7 marks)**

The function $y = \dfrac{1}{(x+2)}$ is drawn below.



(a) On the same axes above, plot the graph of the function $y = 2^{(x+2)}$. (3 marks)

Solution
As shown on graph
Specific Behaviours
✓ identifies y-intercept
✓ identifies behaviour for small x
✓ identifies exponential growth (behaviour for large x)

(b) (i) Using your calculator, solve the equation $2^{(x+2)} = \dfrac{1}{(x+2)}$. (1 mark)

Solution
–1.359 or –1.36
Specific Behaviours
✓ determines solution

(ii) Show on the graph where the answer to (b)(i) can be found. (1 mark)

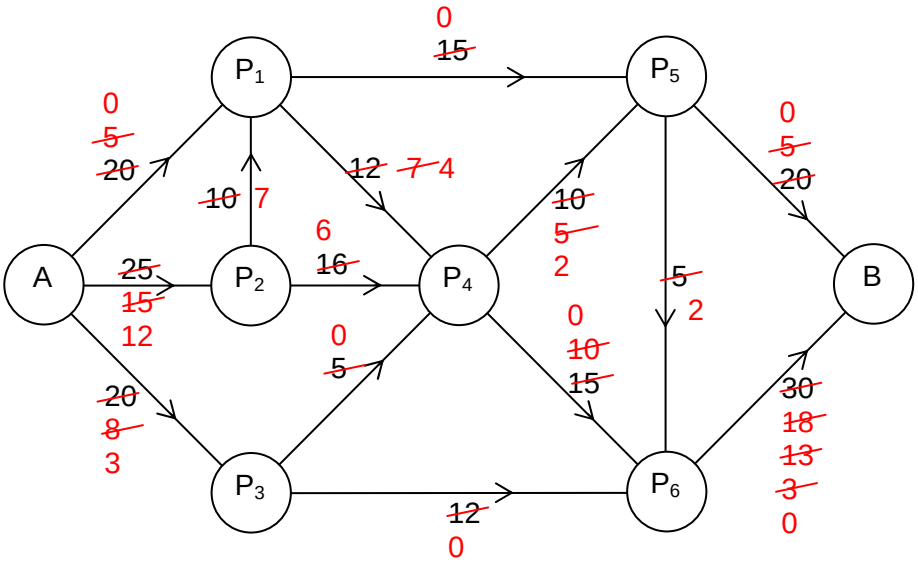
Solution
shown on graph (point B)
Specific Behaviours
✓ identifies correct point

(c) Indicate on the graph the equation $\frac{1}{(x+2)} = -2$ and state the solution. (2 marks)

Solution
Shown on graph (dashed line) x = –2.5 Point C on the graph
Specific Behaviours
✓ identifies point on graph ✓ state correct solution

Question 14 **(5 marks)**

In a waste treatment system, liquid waste is moved from treatment plant A to treatment plant B through a pipeline network containing six pumping stations P_1, P_2, P_3, P_4, P_5 and P_6. The network is displayed below. The number on each arc represents the maximum amount of waste, in tonnes per hour, that can be moved along that pipe segment.



(a) What is the maximum hourly amount of liquid waste that can be moved from treatment plant A to treatment plant B? Show systematic working to allow your solution to be checked. (3 marks)

Solution
A P_1 P_5 B : 15 A P_1 P_4 P_5 B : 5 A P_2 P_1 P_4 P_5 P_6 B : 3 A P_2 P_4 P_6 B : 10 A P_3 P_4 P_6 B : 5 A P_3 P_6 B : 12 50 Maximum flow = 50 tonnes/hour (Alternatives exist, but max flow = 50 tonnes/hr).
Specific Behaviours
Carries through to a degree of accuracy ✓ shows at least 3 flow changes ✓ shows all flow changes ✓ calculates total maximum flow.

(b) What effect, if any, would there be on the maximum flow of liquid waste from A to B if the capacity of the link between P_6 and B was increased by 5 tonnes per hour. Justify your answer. (2 marks)

Solution
Flow would increase by 2 tonnes/hour Through path: A P_2 P_1 P_4 P_5 P_6 B (Note that P_5P_6 has only a capacity of 2). (Follow through from (a)).
Specific Behaviours
✓ identifies effect of change on network flow ✓ describes effect of changed conditions.

Question 15 **(7 marks)**

Table 1 shows some summary statistics for maximum daily temperatures in October 2008 and October 2009 for a Western Australian town. The maximum daily temperatures (°C) in October 2008 for the town are summarised in Table 2.

Table 1: Maximum daily temperatures (°C), October 2008–2009

	October 2008	October 2009
Mean	**20.9**	21.8
Median	20.0	20.7
Standard deviation	**4.4 (4.39)**	4.9
Range	17.0	21.8

Table 2: Maximum daily temperatures (°C), October 2008

Temperature T (°C)	Frequency
$14 \leq T < 18$	9
$18 \leq T < 22$	12
$22 \leq T < 26$	5
$26 \leq T < 30$	4
$30 \leq T < 34$	1

(a) Use the data in Table 2 to:

(i) calculate the mean and standard deviation temperatures for October 2008 and enter the results in Table 1. (2 marks)

Solution
shown above
Specific Behaviours
✓ calculates the mean ✓ calculates the standard deviation

(ii) determine the modal class. (1 mark)

Solution
Modal class is $18 \leq T < 22$
Specific Behaviours
✓ determines the modal class

(b) In which of the two years were the October temperatures in the town less variable? Justify your answer. (2 marks)

Solution
The temperatures in October 2008 were less variable because the standard deviation of 2008 was lower than the standard deviation for 2009. (Range may be mentioned, but standard deviation must be mentioned).
Specific Behaviours
✓ recognises sample with higher standard deviation ✓ correct conclusion.

(c) (i) Tick the box next to the statement that is supported by the data for this Western Australian town. (1 mark)

☐ The year 2008 tended to be cooler than the year 2009.

☐ The year 2008 tended to be hotter than the year 2009.

✓ It is not possible to tell whether the year 2008 tended to be cooler or hotter than the year 2009.

Solution
as shown above.
Specific Behaviours
✓ identifies correct statement

(ii) Justify your choice. (1 mark)

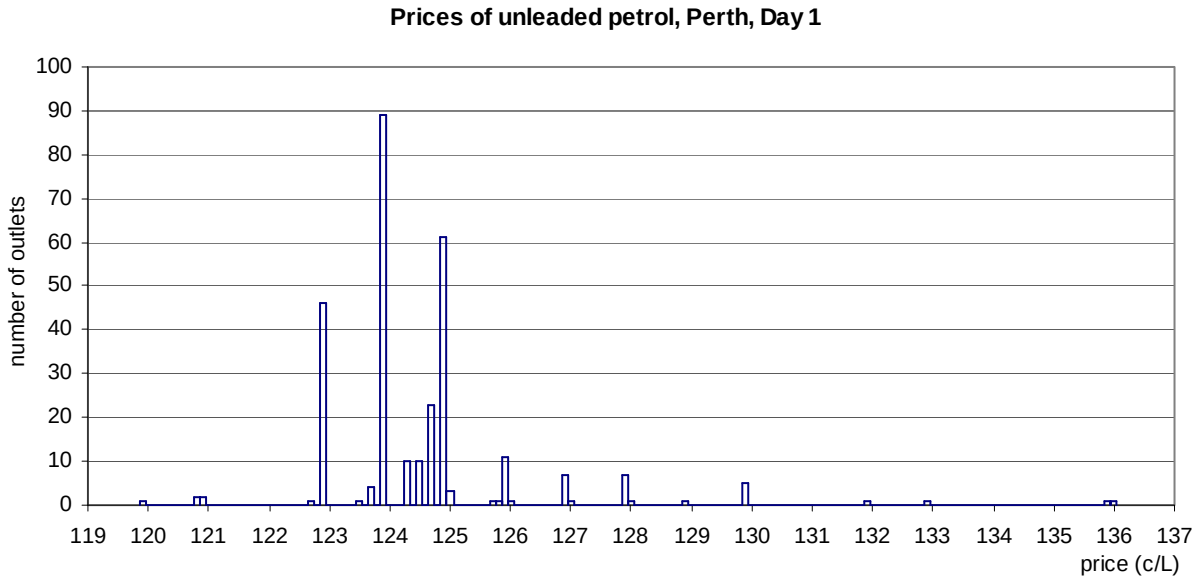
Solution
conclusion about temperature for the year can not be drawn from data for 1 month.
Specific Behaviours
✓ recognises conclusion about yearly temperature cannot be made from 1 month of data.

Question 16 **(12 marks)**

The table shows prices (c/L) of standard unleaded petrol from 293 outlets in Perth on one day in early 2010 (Day 1). All prices published for the day on the Western Australian Government's 'FuelWatch' website are included.

Price (c/L)	Frequency
119.9	1
120.8	2
120.9	2
122.7	1
122.9	46
123.5	1
123.7	4
123.9	89
124.3	10
124.5	10
124.7	23
124.9	61
125.0	3
125.7	1
125.7	1
125.9	11
126.0	1
126.9	7
127.0	1
127.9	7
128.0	1
128.9	1
129.9	5
131.9	1
132.9	1
135.9	1
136.0	1

The graph displays the price data from the table.



(a) Describe the centredness and spread of the prices. Use information in the table and graph. Do not calculate any statistics. (4 marks)

Solution
Centredness: Most prices were close to (or slightly more or less equal to) (or within 1 cent) of 123.9C/L; or in the range 122.9 c/L to 124.9 c/L. Spread: Prices ranged from 119.9 c/L and 136 c/L or two outlets charged more than others (135.9 c/L, 136 c/L) or range in prices was 16.1 c/L or a spread property related to frequency with frequency quantified e.g. the high majority of prices were 124.9 c/L or less (85%); 125.9 c/L or less (91%). Two spread properties.
Specific Behaviours
✓ recognises that most prices were close to "centre" ✓ stating a 'central value' or 'interval' ✓✓ recognises two spread properties

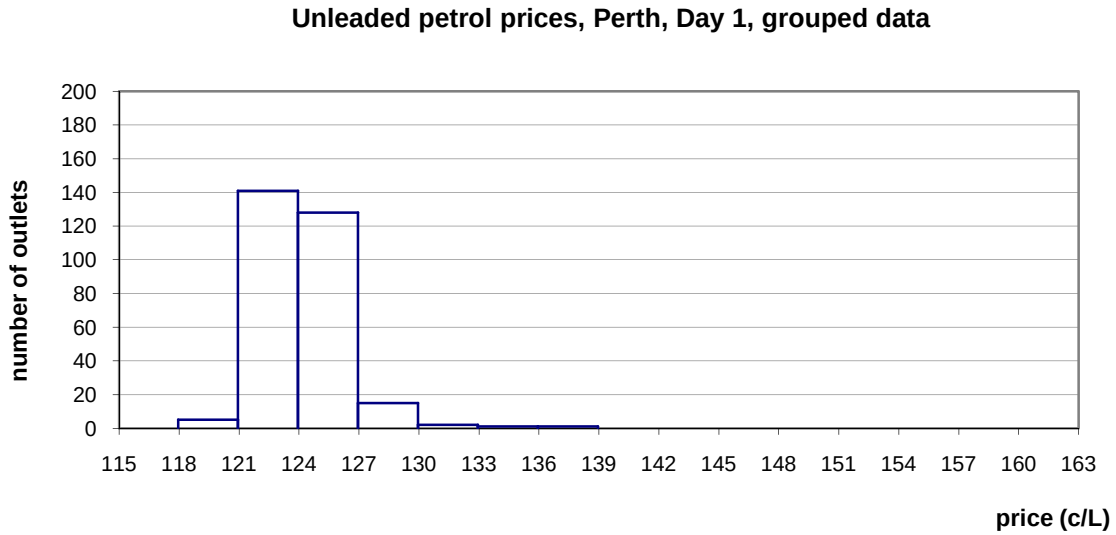
(b) The table below shows the price data grouped in the equal-sized intervals. Complete the table. (2 marks)

Unleaded petrol prices, Perth, Day 1

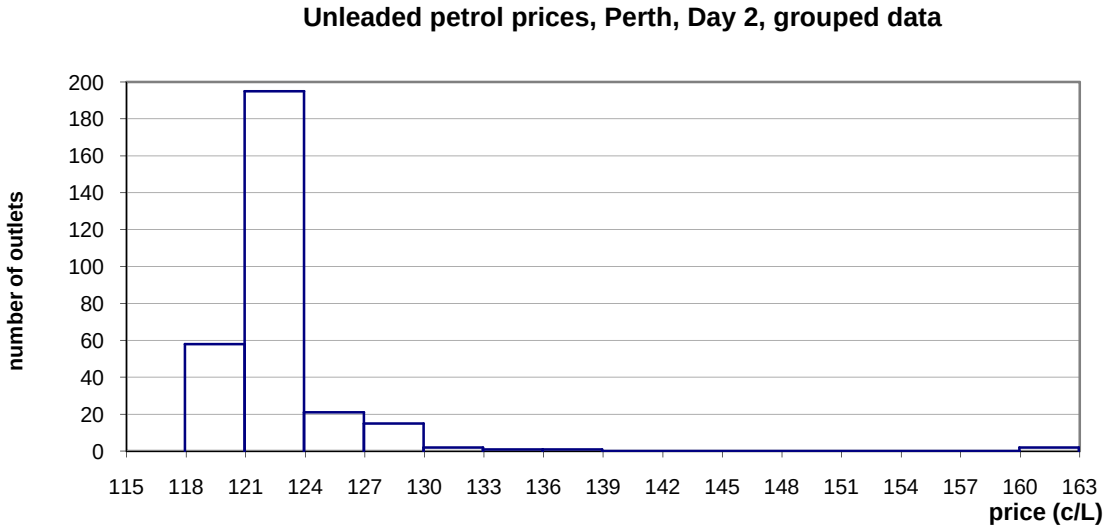
Price (c/L)	Frequency (number of outlets)
118–120.9	**5**
121–123.9	141
124–126.9	128
127–129.9	**15**
130–132.9	2
133–135.9	1
136–138.9	1

Solution
As above
Specific Behaviours
✓✓ determines correct entries.

The frequency histogram for the grouped data for Perth, for Day 1 is shown below.



Unleaded petrol prices from the same outlets were also recorded for the following day (Day 2). The frequency histogram for Day 2 is given below.



(c) Did prices for Day 2 tend to be higher or lower than prices for Day 1? Justify your answer. (4 marks)

Solution
The graph suggests that the prices on Day 2 tend to be lower than the prices for Day 1, because the prices are concentrated (have high frequency/relative frequency) in the intervals 118–120.9 c/L and 121.0 - 123.9 c/L for Day 2 and in the intervals 121.0–123.9 c/L and 124.0–126.9 c/L for Day 1.
Specific Behaviours
✓ identifies correct price behaviour ✓ gives reasonable explanation for choice ✓ identifies modal interval ✓ identifies another interval with high frequency

(d) The data for Day 2 includes a price of 161.2 c/L. If this value is removed from the data set, describe the effect on the

(i) mean price value. (1 mark)

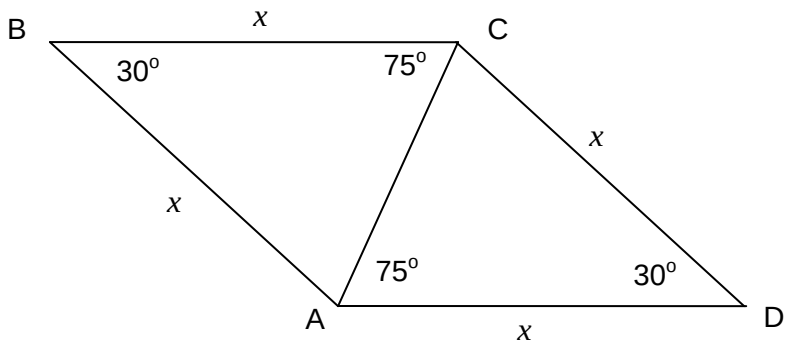
Solution
Mean will decrease
Specific Behaviours
✓ identifies removing the outlier will reduce the mean price

(ii) standard deviation value. (1 mark)

Solution
Standard deviation will decrease
Specific Behaviours
✓ identifies removing the outlier will reduce the spread, including the standard deviation

Question 17 **(8 marks)**

The diagram below shows a plan for an indoor sports centre ABCD that is to be built in the shape of a rhombus with side length x metres. A walkway is planned from A to C.



(a) Write an expression for the distance between A and C. (2 marks)

Solution
Distance = $\sqrt{x^2 + x^2 - 2 \times x \times x \cos 30^\circ}$ metres.Alternatives: .5176x;2xsin15^0 ;xsin30^0/sin75^0 ;
Specific Behaviours
✓ applies correct values to cosine rule ✓ includes square root in expression

(b) Write a simplified expression for the total floor area of the sports centre. (3 marks)

Solution
Area ABCD = $\left(\frac{1}{2} \times x \times x \sin 30^\circ\right) \times 2$ Area ABCD = $\left(\frac{1}{2}x^2 \times \frac{1}{2}\right) \times 2 = \frac{1}{2}x^2$ m^2
Specific Behaviours
✓ applies correct rule for area of a triangle ✓ multiplies area formula by two for quadrilateral ✓ simplifies expression

A decision has been made to increase all sides of the rhombus by 20%.

(c) (i) Give an expression for the total floor area of the sports centre in the form $ax^2 + bx + c$. (2 marks)

Solution
Area ABCD = $\left(\frac{1}{2} \times 1.2x \times 1.2x \sin 30^\circ\right) \times 2$ Area ABCD = $0.72x^2$ m^2
Specific Behaviours
✓ determines an expression for the area of the quadrilateral (using side x 1.2)
✓ simplifies expression for area to the form $ax^2 + bx + c$

(ii) How will increasing the sides of the rhombus effect the total floor area of the sports centre? Show your working. (1 mark)

Solution
$0.72x^2 \div \frac{1}{2}x^2 = 1.44$ $\therefore$ 44% increase in the total floor area
Specific Behaviours
✓ determines the effect of increasing the side lengths

Question 18 **(5 marks)**

The table below shows three points of a linear function.

x	1	2	3
y	$3n$	$4n$	$2n + 18$

(a) Determine the value of n. (2 marks)

Solution
$4n - 3n = 2n + 18 - 4n$ $n = 6$
Specific Behaviours
✓ recognises the common difference ✓ calculates value of n.

(b) Hence determine the equation of the linear rule for this function. (2 marks)

Solution
Terms are 18, 24, 30 common difference 6. $y = 6x + 12$
Specific Behaviours
✓ determines the gradient ✓ determines the y-intercept

(c) Comment on the link between the gradient of your linear rule in (b) and the recursive rule that defines the T_n values in the table below. (1 mark)

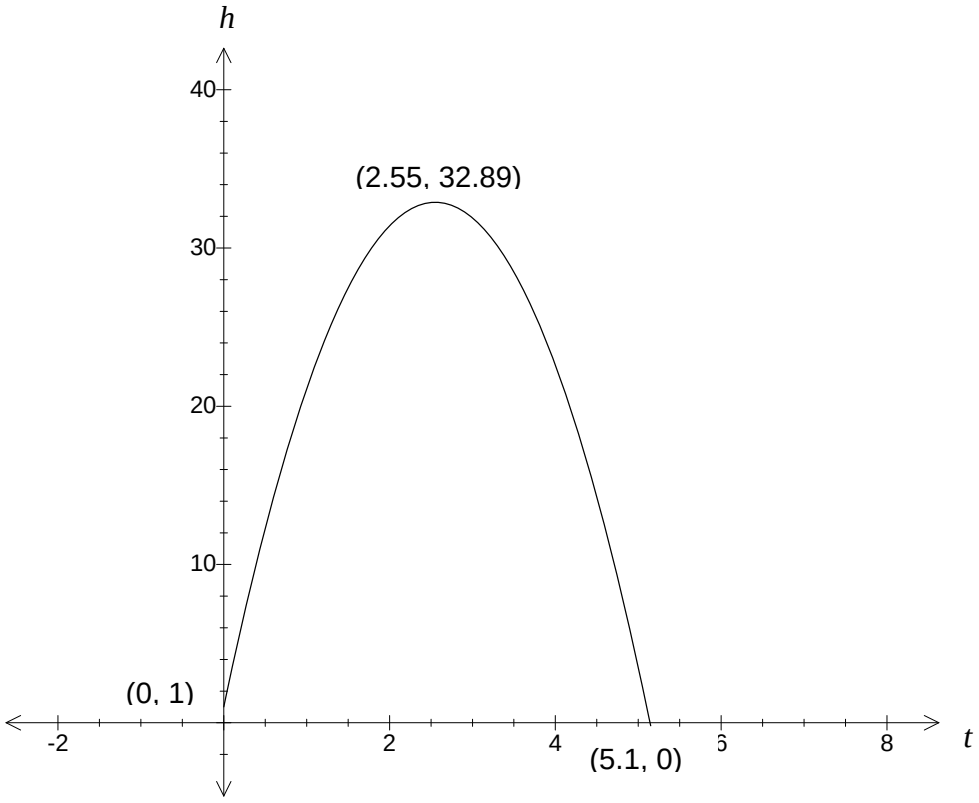
n	1	2	3
T_n	$3n$	$4n$	$2n + 18$

Solution
$T_{n+1} = T_n + 6, T_1 = 18$ The common difference in recursive rule = gradient in linear rule
Specific Behaviours
✓ describes the link

Question 19 **(6 marks)**

A ball machine sitting on level ground is projecting baseballs into the air so that baseballers can practise their outfield catches. The height (h), in metres, is given by $h = 25t - 4.9t^2 + 1$, where t is the time in seconds after projection.

(a) Draw the path of the ball on the axes below, labelling all key features. (4 marks)



Solution
shown above
Specific Behaviours
✓ shows shape (graph within bounds, i.e. $t \geq 0$ and $h \geq 0$)
✓ identifies t-intercept at (5.1, 0)
✓ identifies h-intercept at (0, 1)
✓ identifies maximum at (2.55, 32.89)

(b) Determine the length of time that the ball is at least 14 metres above the ground.
(2 marks)

Solution
$25t - 4.9t^2 + 1 = 14$
$t = 0.59, t = 4.51$
$t = 3.92$ seconds
Specific Behaviours
✓ solves $25t - 4.9t^2 + 1 = 14$
✓ determines time difference

ACKNOWLEDGEMENTS

Section Two

Question 9 Data source: Australian Taxation Office. (2010). *Individual income tax rates*: *2009–10*. Retrieved March 2010, from www.ato.gov.au/individuals/content.asp?doc=/content/12333.htm.

Queston 11 Data source: Western Australian Office of Road and Safety. (n.d.). *Number of road crashes in Western Australia 1995–2004, by days of the week.* Retrieved March 2010, from http://www.ors.wa.gov.au.

Question 15 Data source: Australian Government. Bureau of Meteorology. (n.d.). Retrieved March, 2010, from http://www.bom.gov.au/.

Question 16 Data source: Government of Western Australia. Department of Commerce. *Fuel Watch*. Retrieved March 2010, from http://www.fuelwatch.wa.gov.au.